

THE KELVIN-HELMHOLTZ INSTABILITY IN ORION:
A SOURCE OF TURBULENCE AND CHEMICAL MIXINGO. BERNÉ^{1,2}, Y. MATSUMOTO³,
Draft version October 25, 2012

ABSTRACT

Hydrodynamical instabilities are believed to power some of the small scale (0.1-10 pc) turbulence and chemical mixing in the interstellar medium. Identifying such instabilities has always been difficult but recent observations of a wavelike structure (the *Ripples*) in the Orion nebula have been interpreted as a signature of the Kelvin-Helmholtz instability (KHI), occurring at the interface between the H II region and the molecular cloud. However, this has not been verified theoretically. In this letter, we investigate theoretically the stability of this interface using observational constraints for the local physical conditions. A linear analysis shows that the H II /molecular cloud interface is indeed KH unstable for a certain range of magnetic field orientation. We find that the maximal growth-rates correspond to typical timescales of a few 10^4 years and instability wavelengths of 0.06 to 0.6 pc. We predict that after 2×10^5 years the KHI saturates and forms a turbulent layer of about 0.5 pc. The KHI can remain in linear phase over a maximum distance of 0.75 pc. These spatial and time scales are compatible with the *Ripples* representing the linear phase of the KHI. These results suggest that the KHI may be crucial to generate turbulence and to bring heavy elements injected by the winds of massive stars in H II regions to colder regions where planetary systems around low mass stars are being formed. This could apply to the transport of ^{26}Al injected by a massive star in an H II region to the nascent solar-system.

Subject headings: ISM: kinematics and dynamics, Magnetohydrodynamics (MHD), Instabilities, Astrochemistry

1. INTRODUCTION

Over 50 years ago, it was postulated by Spitzer (1954) and Frieman (1954) that hydrodynamical instabilities may form in interface regions between the hot diffuse gas ionized by massive stars and cold dense molecular clouds. In particular, these authors suggested that *elephant trunk*, or spike structures, which are widely observed in star-forming regions, may result from the Rayleigh-Taylor (see Chandrasekhar 1961) instability (RTI). Another classical type of interface instability is the Kelvin-Helmholtz instability (see Chandrasekhar 1961) which occurs in the presence of a velocity shear across the interface and is characterized by a wave-like periodic structure. Both types of instabilities have been considered to play a dominant role in the interstellar medium, as a power source for small scale (0.1-10 pc) turbulence (Elmegreen 2004) and in the mixing of chemical elements (Roy & Kunth 1991). Observationally, it has been hard to confirm the existence of these instabilities. Although the observed sizes of *elephant trunks* match first order theoretical models of RTI (Frieman 1954), recent observations of the velocity field in the Pillars of Creation and the Horsehead nebula by Pound (1998) and Pound, Reipurth & Bally (2003) tend to discard the RTI hypothesis for the formation of these structures (instead, the selective photodissociation model of Reipurth (1983) is invoked). More recently, Berné, Marcelino & Cernicharo (2010) (BMC hereafter)

observed a periodic wavelike structure (the *Ripples* hereafter), at the surface of the Orion cloud (Fig. 1) which appears to be compatible with a KHI. However, this work was mostly qualitative and lacked a detailed model to assess if the development of the KHI is possible in conditions as those found in Orion. In addition, this former study did not consider the possible effect of magnetic fields—which are known to be strong in Orion (Abel et al. 2004)—on the KHI. Finally, even if there is evidence for the existence of the KHI in the interstellar medium (ISM), it remains unclear over which timescale this instability may convert energy into turbulent motion of the gas and hence if it can actually play a role in mixing the hot and cold gas. On the theoretical side, extensive numerical magnetohydrodynamical (MHD) models for the KHI have been developed and successfully applied to explain observed phenomena in solar-system plasmas (e.g. Matsumoto & Hoshino 2004). In addition, Matsumoto & Seki (2010) have studied in great details the saturation of the KHI and its evolution into turbulence and mixing of the gas. In this letter, we perform a linear MHD analysis applied to the situation in Orion's *Ripples*, using physical parameters determined observationally. We derive the key parameters that characterize the instability and use them to determine the timescales over which the gas becomes turbulent due to the saturation of the instability. We discuss these results in the context of chemical mixing in the interstellar medium and transport of ^{26}Al in the solar system.

2. OBSERVATIONS

Fig. 1 shows the *Spitzer* Space Telescope (Werner et al. 2004) mid-infrared (mid-IR) image of the *Ripples* in Orion, which have been attributed to a KHI by BMC.

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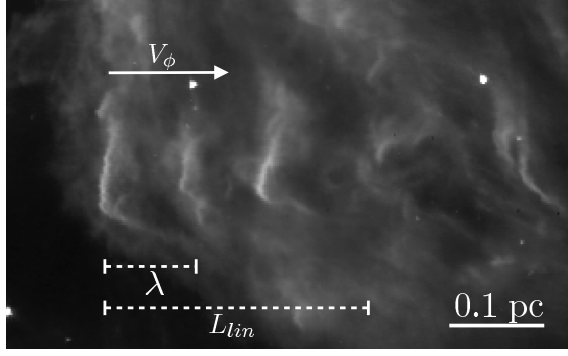


Figure 1. *Spitzer* Infrared Array Camera (Fazio et al. 2004) 8 μm image of the *Ripples* in Orion which have been attributed to a KHI by BMC. λ denotes the spatial wavelength of the structure, L_{lin} is the distance over which the instability appears to be completely linear. The orientation of V_ϕ , the phase velocity of the KH wave at the surface of the cloud, is depicted qualitatively.

KHIs occur at the interface between two fluids flowing relative to each other. In the case of Orion, the two fluids are the H II gas and the neutral gas of the molecular cloud, and the velocity shear results from the *champagne flow* created by the H II region bursting through the parental molecular cloud. When growing, the KHI gives to the interface a wavelike structure which can be seen in Fig. 1. An important parameter of the KHI is its spatial wavelength λ which is connected to the physical conditions in which the instability occurs. Here, λ can be measured directly from the image and is found to be $\lambda = 0.11$ pc for a distance to the Orion nebula of 414 pc. This corresponds to a spatial wavenumber $k = 2\pi/\lambda = 2 \times 10^{-15} \text{m}^{-1}$.

3. LINEAR ANALYSIS OF THE KELVIN HELMHOLTZ INSTABILITY IN ORION

3.1. Objectives and method

Our goal here is to study from the theoretical point of view, and using realistic physical conditions, the stability of an H II / molecular cloud interface against the KHI. In particular we want to determine whether magnetic fields can play a stabilizing role. It is also of great interest to derive some of the key parameters, for instance the growth rate γ and the range of acceptable wavelengths. In order to do this, we perform a linear stability study which includes magnetic field, compressibility and an analytical velocity profile across the sheared layer (see below). This differs greatly from the preliminary study of BMC which relied on an ideal case (Chandrasekhar 1961) of incompressible fluids with a discontinuous velocity profile and no magnetic field. The present study is performed in a two-dimensional slab geometry, described in Fig. 2 for the initial conditions. These initial conditions are maintained by the magnetohydrodynamical equilibrium. The density gradient and velocity gradients are along the y axis. The velocity and density profiles across the interface are of hyperbolic-tangent form (Miura & Pritchett 1982). The velocity is oriented along the x axis. The magnetic field direction is inside the plane defined by y and z and its orientation is defined by the angle θ between \vec{B} and z . For this first analysis, we have not considered the azimuthal dependence for the

orientation of \vec{B} , because this would imply heavy complications in the solving of the MHD equations. For the adopted configuration, the MHD equations are linearized and a perturbed quantity f can be expanded as a plane wave in the form of $f(x, y) = \hat{f}(y) \exp\{i(k_x x - \omega t)\}$. The linearized equations can then be solved, with boundary conditions in the y direction and a given wave number k_x for the corresponding eigen value (angular frequency and growth rate) as an eigen value problem. This is described in mathematical terms in the following section.

4. LINEAR MODEL

4.1. Basic equations

The basic MHD equations are

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{V}), \quad (1)$$

$$\frac{\partial \mathbf{V}}{\partial t} = -(\mathbf{V} \cdot \nabla) \mathbf{V} - \frac{1}{\rho} \nabla (P + \frac{B^2}{8\pi}) + \frac{1}{4\pi} (\mathbf{B} \cdot \nabla) \mathbf{B}, \quad (2)$$

$$\frac{\partial P}{\partial t} = -(\mathbf{V} \cdot \nabla) P - \Gamma P (\nabla \cdot \mathbf{V}), \quad (3)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -c \nabla \times \mathbf{E}, \quad (4)$$

with the frozen-in condition

$$\mathbf{E} = -\frac{\mathbf{V}}{c} \times \mathbf{B}. \quad (5)$$

where $\Gamma = 5/3$ is a polytropic constant. The mass density ρ and the magnetic field \mathbf{B} are normalized by characteristic values of ρ_0 and B_0 , the velocity \mathbf{V} by the velocity jump across the boundary, V_0 , the pressure P by $B_0^2/8\pi$, the spatial scale by the initial shear width L , and the time by L/V_0 .

4.2. Linearization and solution

We consider perturbed quantities from an equilibrium state as,

$$\begin{aligned} \mathbf{V} &= \mathbf{V}_0 + \delta \mathbf{v}, \\ P &= P_0 + \delta p, \\ \mathbf{B} &= \mathbf{B}_0 + \mathbf{b}. \end{aligned}$$

The perturbed quantities are expressed as a plane wave in the form, $\delta A = \tilde{A}(y) \exp[i(k_x x - \omega t)]$, where A , k_x and $\omega = \omega_r + i\gamma$ denote a physical parameter, the wave number in the x direction, and the angular frequency, respectively.

Linearizing the above MHD equations (1-4), one obtains

$$\begin{aligned} \omega \delta v_x &= k_x V_x \delta v_x - i \frac{\delta V_x}{\partial y} \delta v_y + \frac{k_x}{2n_0} \delta p + k_x \frac{B_{0z}}{n_0} b_z + \frac{i}{n_0} \frac{\partial B_{0x}}{\partial y} b_y, \\ \omega \delta v_y &= k_x V_x \delta v_y - \frac{i}{2n_0} \frac{\partial}{\partial y} (\delta p + 2\mathbf{B}_0 \cdot \mathbf{b}) - k_x \frac{B_{0x}}{n_0} b_y, \\ \omega \delta v_z &= k_x V_x \delta v_z - k_x \frac{B_{0x}}{n_0} b_z + \frac{i}{n_0} \frac{\partial B_{0z}}{\partial y} b_y, \\ \omega \delta p &= k_x V_x \delta p - i \delta v_y \frac{\partial P_0}{\partial y} + \Gamma P_0 k_x \delta v_x - i \Gamma P_0 \frac{\partial \delta v_y}{\partial y}, \end{aligned}$$

$$\begin{aligned}
\omega b_x &= k_x V_x b_x - i B_{0x} \frac{\partial \delta v_y}{\partial y} - i \delta v_y \frac{\partial B_{0x}}{\partial y} + i b_y \frac{\partial V_x}{\partial y}, \\
\omega b_y &= k_x V_x b_y - k_x B_{0x} \delta v_y, \\
\omega b_z &= k_x V_x b_z + k_x B_{0z} \delta v_x - i B_{0z} \frac{\partial \delta v_y}{\partial y} - i \delta v_y \frac{\partial B_{0z}}{\partial y} - k_x B_{0x} \delta
\end{aligned}$$

Discretizing the spatial derivative of the Fourier amplitude in the y direction with boundary conditions at $y = \pm y_b$ far away from the shear layer, the equations can be cast into the form of an eigen value problem as

$$\omega \begin{pmatrix} \delta v_x \\ \delta v_y \\ \delta v_z \\ \delta p \\ b_x \\ b_y \\ b_z \end{pmatrix} = M \begin{pmatrix} \delta v_x \\ \delta v_y \\ \delta v_z \\ \delta p \\ b_x \\ b_y \\ b_z \end{pmatrix}, \quad (7)$$

from which we can obtain the eigen values of ω , whose imaginary part is the growth rate, and the corresponding eigen functions for any k_x . To solve this eigen value problem, we use the QR algorithm (Francis 1961, 1962; Kublanovskaya 1963).

4.3. Adopted initial parameters

Solving the above described problem requires to know the initial conditions of the set-up. Most of them can be derived from observations, or estimated. The adopted physical conditions are summarized in Table 1. The density and temperature (n_I, T_I) of the molecular cloud have been discussed in BMC. According to their results we adopt $n_I = 10^4 \text{ cm}^{-3}$ and $T_I = 20 \text{ K}$. For the H II region, we consider a typical temperature $T_{II} = 10^4 \text{ K}$, and $n_{II} = 20 \text{ cm}^{-3}$. The velocity shear is taken to be $V_0 = 10 \text{ km.s}^{-1}$ following Roy & Kunth (1991), also typical for such environments. Magnetic field strength has been measured in the Orion nebula by Brogan et al. (2005) and Abel et al. (2004) and found to vary between 5 nT for the Trapezium region and 25 nT in the Veil region. The *Ripples* are likely situated between the Trapezium and the Veil so we adopt a conservative value of $B = 20 \text{ nT}$. The value of θ cannot be determined independently so we have considered various values between 0 and 90° (with symmetrical results in the -90 to 0° range). Finally, BMC derived a gravitational field at the cloud surface $g = 3.5 \times 10^{-11} \text{ m s}^{-2}$. We can compare the potential to kinetic energy using the Richardson number $R_i = g L / (V_0)^2$ (Chandrasekhar 1961). The value of L can be estimated to be the thickness of the photodissociation region (PDR) measured by BMC, which is essentially the region where the gas is converted from fully neutral to fully ionized and where the temperature changes from a few 10 K to a few 1000 K (Tielens & Hoellenbach 1985). L is typically 0.01 pc (BMC), hence $R_i \sim 10^{-3}$ which implies that gravity is ineffective and it is no further included in the calculations.

5. RESULTS

5.1. Linear phase of the KHI

The results of the linear analysis concern the linear growth of the instability. They are presented in Fig. 3

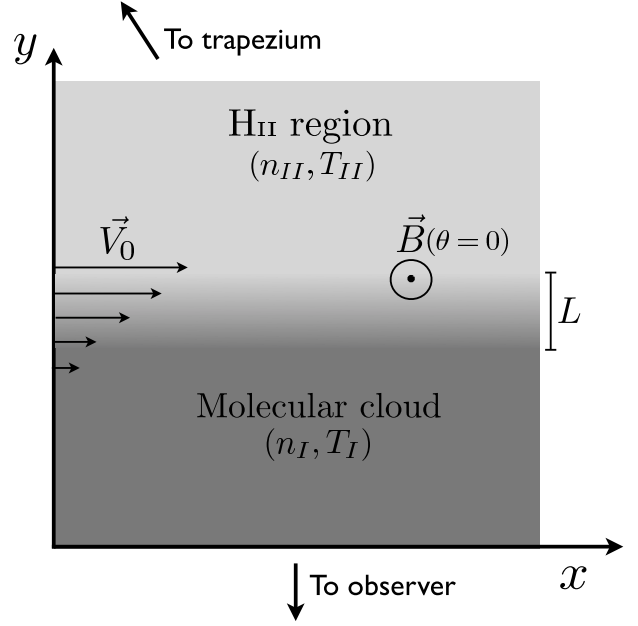


Figure 2. Schematic representation of the configuration adopted for the initial conditions of the interface between the H II region and the molecular cloud.

and Fig. 4. Fig. 3 shows the influence of the value of θ , the inclination of the magnetic field, on the normalized growth rate $\gamma L / V_0$ and on the normalized wavenumber $k_x L$. The growth rate decreases with increasing angle but for $-25^\circ \lesssim \theta \lesssim 25^\circ$ the growth rate is non-zero, implying that the interface is KH unstable. In this range of values for θ , the wavenumber k_x is almost constant, showing that it does not depend on magnetic field orientation. Fig. 4 shows the evolution of the normalized growth rate $\gamma L / V_0$ as a function of the normalized wavenumber $k_x L$, for $\theta = 0$. The most unstable mode corresponds to $k_x L = 0.56$, and typically, for $0.1 < k_x L < 1$ the growth rate is high and corresponds to an e-fold timescale of less than 10^5 years, short compared to the lifetime of an OB association (~ 10 Myrs). Therefore, it is expected that instabilities with $0.1 < k_x L < 1$ appear in star-forming regions. Using $L = 0.01 \text{ pc}$, imposes that the wavelength of the instability λ_{KH} will range between 0.06 and 0.6 pc. This number also places limits on the detectability of KHIs: at a distance of 1 kpc this is an angular size of 4-40", and at 10 kpc this is 0.4-4". Hence, KHI structures are expected to be of small angular size and can only be observed with high angular resolution telescope and/or in nearby regions of massive star formation like Orion.

5.2. Saturation of the KHI

Matsumoto & Seki (2010) studied in details the 2 dimensional evolution of the KHI for conditions similar to those presented here and found that the growth of the instability leads to saturation. This results in the formation of a turbulent layer where the two fluids are mixed, over a time-scale of the order of $t_{sat} \sim 200 L / V_0$. Hence, using the observed value for L (Table 1) this results in a saturation timescale $t_{sat} \sim 2 \times 10^5$ years. It is important to realize that even if t_{sat} is short there is always

Table 1
Physical parameters

Parameter	Symbol	Reference
Observational		
Heliocentric distance	d	414 pc (1)
Neutral gas density	n_I	10^4 cm^{-3} (2)
Ionized gas density	n_{II}	20 cm^{-3} (2)
Neutral gas Temperature	T_I	20 K (3)
Ionized gas Temperature	T_{II}	10^4 K (4)
Velocity sheer adopted here	V_0	10 km.s^{-1} (5)
Gravitational field	g	$3.5 \times 10^{-11} \text{ m.s}^{-2}$ (2)
Magnetic field strength	B	20 nT (6)
Magnetic field orientations	θ	$0-90^\circ$
Instability wavelength	λ	0.11 pc (2)
Width of the sheered layer	L	0.01 pc (2)
Linear regime length	L_{lin}	0.3 pc Fig. 1
Derived values (for maximal growth rate)		
Instability growth rate	γ	$2.3 \times 10^{-13} \text{ s}^{-1}$
Instability phase velocity	V_ϕ	3.6 km.s^{-1}
Instability saturation timescale	t_{sat}	$2 \times 10^5 \text{ yrs}$
Size of mixing layer after t_{sat}	L_{mix}	0.5 pc
Distance travelled before saturation	L_{sat}	0.74 pc

(1) From Menten et al. (2007), (2) From BMC, (3) Typical for molecular clouds (Tielens 2005), (4) Typical for H II regions (Tielens 2005), (5) From Roy & Kunth (1991), (6) Based on Abel et al. (2004)

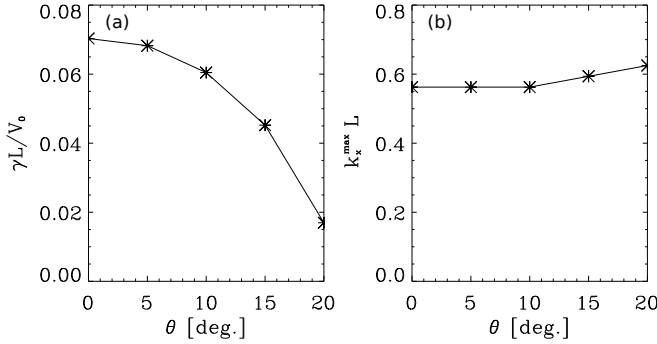


Figure 3. *Left:* Normalized growth rate $\gamma L/V_0$, of the most unstable mode, as a function of the orientation of the magnetic field. *Right:* Normalized spatial wavenumber $k_x L$ of the most unstable mode as a function of the orientation of the magnetic field.

a part of the instability that remains linear. According to the results of our linear analysis we find that the KH mode travels along the boundary layer with speed of $V_\phi = 0.36 \times V_0$, that is $\sim 3.6 \text{ km/s}$. Hence, we can define L_{sat} , the spatial scale before the instability has saturated by $L_{sat} = V_\phi t_{sat}$ and find $L_{sat} \sim 0.74 \text{ pc}$. These theoretical results have several implications. First, the timescale for saturation is short compared to the lifetime of an OB association, so KHIs will saturate and will be a source of turbulence. Secondly, the scale size over which it is possible to see the linear regime is *at maximum* 0.74 pc.

6. THE RIPPLES AS A KHI

Based on the theoretical results described above, we investigate in more details if the *Ripples* can be interpreted as an occurrence of the linear phase of the KHI. The observed value of λ for the *Ripples* is 0.1 pc, which fall in the range defined from the theoretical investigation ($0.06 < \lambda_{KH} < 0.6 \text{ pc}$). The *Ripples* seem to preserve a very periodic structure, suggesting linear regime, over a distance $L_{lin} = 3\lambda = 0.3 \text{ pc}$ (Fig. 1). The following 2 bil-

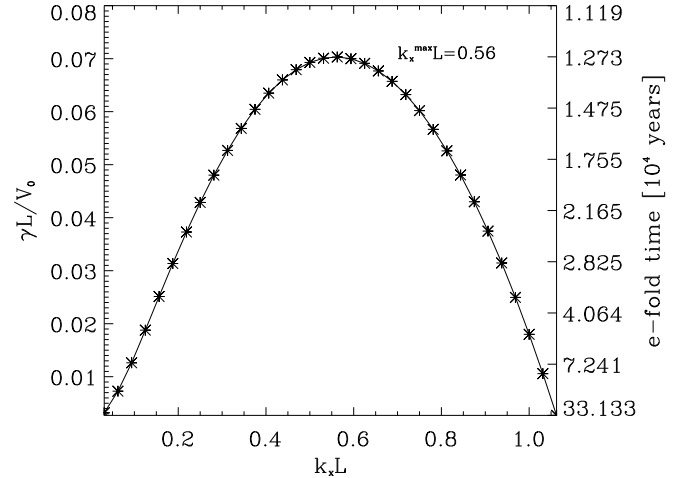


Figure 4. Normalized growth rate $\gamma L/V_0$ versus normalized spatial wavenumber $k_x L$ of the instability. The right axis translates the value of the growth rate into an e-fold timescale for Orion.

lows, instead, start showing some chaotic structure suggesting the beginning of the saturation of the instability. After about 5λ , the periodic structure has disappeared. Hence, the travelled distance in linear regime L_{lin} for the *Ripples* is, consistently with the KHI model, smaller than the maximal theoretical value of $L_{sat} = 0.74 \text{ pc}$ discussed in Sect. 5. Therefore, we argue that the observed evolution of the *Ripples* structure results from the motion of the KH wave towards saturation, from left to right in Fig. 1. Our last remark concerns the importance of the magnetic field. As mentioned above the *Ripples* can only result from a KHI if they are in a region of small θ . Orientation of the magnetic field lines in Orion have been measured by e.g. Houde et al. (2004); Poidevin, Batien & Matthews (2010) using polarimetry. Unfortunately, this does not cover the *Ripples* region

and, in addition, performing such measurements at the arc-second scale remains challenging. We can only stress the importance –often neglected– that magnetic fields have in shaping the interstellar medium, in this case because they can stabilize interfaces between H II regions and molecular clouds.

Altogether, we conclude that the study presented in this letter brings additional evidence that the *Ripples* result from a KHI. This raises one question however, which is “why do we only see one occurrence of the KHI in Orion?” This is perhaps because only in this region are the conditions (e.g. magnetic field, velocity flow) favorable at the moment we observe Orion. Over the lifetime of the region however, this may occur a high number of times. Ripples have indeed been observed recently in an other star forming region (Cygnus OB2) by Sahai et al. (2012), suggesting that the structure in Orion is not unique. It is also possible that more of these structures exist in Orion, but given their small angular size or unfavorable orientation on the plane of the sky they remain undetectable (see Sect. 5).

7. THE KHI AND CHEMICAL MIXING IN STAR-FORMING REGIONS

7.1. Saturation of the instability: turbulence and chemical mixing

We have discussed in Sect. 5 the saturation of the KHI towards a turbulent regime. This may have an important role in chemical mixing in star forming regions. Roy & Kunth (1991) were the first to recognize the importance of the KHI in chemical mixing of the interstellar medium. They studied the influence of KHIs by defining the e-fold timescale assuming a spatial wavelength of 100 pc (the size of a large H II region), and from this derived a timescale of 1.5×10^6 years, which is smaller than the lifetime of an OB association. However, their model was based on an ideal hydrodynamical case with no magnetic field, for which the wavelength of maximal growth rate cannot be determined. This is partly due to the fact that no observational evidence of KHIs existed at the time. The results presented in this Letter using a more detailed model and guided by direct observations, are clearly incompatible with a maximal growth rate corresponding to $\lambda = 100$ pc. In addition, Roy & Kunth (1991) used the e-fold timescale as a measure of mixing timescale, which is not appropriate. Instead, we can use the results found here and those of Matsumoto & Seki (2010) to obtain some general appreciation of the efficiency of the KHI mixing in star-forming regions. First, as mentioned above, full mixing of the fluids occurs after t_{sat} which we have found to be of the order of a few 10^5 years. Again, this is short compared to the lifetime of an OB association so the process will be efficient. After this time, the size of the mixed layer is $L_{mix} = 50 \times L$ (Matsumoto & Seki 2010 Fig. 9), that is 0.5 pc. All in all, in agreement with Roy & Kunth (1991) (although making different hypotheses), we conclude that the KHI can be an efficient mechanism to mix chemical elements in the interstellar medium.

7.2. Further implications: ^{26}Al in the solar system

It is believed that low-mass star formation is triggered in the over-dense shell of molecular cloud that lie around

H II regions (see e.g. Deharveng et al. 2010). In a recent paper, Gounelle & Meynet (2012) argue that the Solar system may have formed in such an environment, based on the abundances of short lived radionuclides found in meteorites. In particular, they propose that ^{26}Al was brought to the forming solar system by the winds of a massive star (see also Montmerle et al. 2007) rather than by Supernovae. This requires that a nearby massive star ($M_\star > 32M_\odot$) injected ^{26}Al during a few million years, and that this element was then well mixed with the H II gas, and eventually that the H II gas was mixed with the surrounding molecular shell efficiently. In Orion at least, the gas from the wind seems to be well mixed with the H II region as shown by Güdel et al. (2008). The efficiency of H II mixing with the molecular shell was not evaluated by Gounelle & Meynet (2012). However, they derive the time t_\star , during which the molecular shell has to be enriched before the solar system starts to form. This value ranges between 0.65 and 6.2 Myr. The mixing timescale we have derived here ($t_{sat} = 2 \times 10^5$ yrs) is smaller than t_\star so that indeed mixing by the KHI is efficient to enrich the molecular shell with ^{26}Al . Hence, the KHI (and possibly other instabilities) could have played an important role in the transport of ^{26}Al to the forming Solar system.

8. CONCLUSION

We have shown that the KHI develops rapidly at the H II molecular cloud interface in conditions like Orion (which are representative of many massive star forming regions). After travelling at the surface of the cloud during a time of a few 10^5 years and over a maximum distance of ~ 0.74 pc, the instability reaches saturation. Hence, as suspected, the KHI is probably a significant mechanism to generate small scale (< 1 pc) turbulence in molecular clouds near massive stars. In addition, since the H II region is contaminated by the chemical elements injected by massive stars winds, the KHI may be a relevant process to bring these elements inside the molecular cloud, in regions where planetary systems around young stars are formed. This could have been at play to transport ^{26}Al to the nascent solar system. Periodic structures corresponding to the linear phase of the KHI (like the *Ripples*) should be relatively widespread in star-forming regions, for instance on the surface of molecular globules as reported recently (Sahai et al. 2012). However, these structures are expected to be small (few arc-seconds) and hence require high angular resolution observations to be identified.

This work was partly supported by Chiba University. We acknowledge Thierry Montmerle, Pierrick Martin, Tomoyuki Hanawa, and Ryoji Matsumoto for fruitful discussion. O. B. is funded by a CNES fellowship.

REFERENCES

- Abel, N. P., Brogan, C. L., Ferland, G. J., 2004, ApJ, 609, 247
- Berné, O., Marcelino, N. & Cernicharo, J. 2010, Nature, 466, 947
- Brogan, C. L., Troland, T. H., Abel, N. P. 2005 ASP Conference Series, 343, 183
- Chandrasekhar, Hydrodynamic and hydromagnetic stability International Series of Monographs on Physics, Oxford: Clarendon, 1961

- Deharveng, L., Schuller, F., Anderson, L. D. et al 2010 A&A, 523, 6
- Elmegreen, B. & Scalo, J. 2004 ARA&A, 42, 211
- Elmegreen, B. & Lada, C. J. 1977 ApJ, 214, 725
- Fazio, G. G., Hora, J. L., Allen, L. E., 2004, ApJS, 154, 10
- Francis, J. G. F., 1961, The Computer Journal, 4, 265
- Francis, J. G. F., 1962, The Computer Journal, 4, 332
- Frieman, E. A., 1954, ApJ 120, 18
- Gounelle, M. & Meynet, G., 2012, A&A, 545, 4
- Güedel, M., Briggs, K., R., Montmerle, T. et al. (2008), Science, 319, 309
- Houde, M., Dowell, C. D., Hilderbrand, R. H. et al. (2004), ApJ 604, 717
- Kublanovskaya, V. N. 1963, USSR Computational Mathematics and Mathematical Physics, 1, 637
- Matsumoto, Y. & Hoshino, M., GRL, 2004, 31, 2807
- Matsumoto, Y. & Seki, K., JGR, (2010) 115, A10231
- Menten, K. M., Reid, M. J., Forbrich, J., Brunthaler, A., 2007, A&A, 474, 515
- Miura, A. & Pritchett, P. L., 1982, JGR, 87, A9
- Montmerle M., Gounelle, M., Guedel, M., et al. (2007) Workshop on the Chronology of Meteorites and the Early Solar System
- Poidevin, F., Bastein, P., Matthews, B. C., (2010) ApJ, 716, 893
- Pound, M. W., Reipurth, B., Bally, J. 2003, AJ, 125, 2108
- Pound, M. W., 1998, ApJL, 493, 113
- Reipurth, B., 1983, A&A, 117, 183
- Roy, J.-R., Kunth, D. 1995, A&A, 294
- Sahai, R., Morris, M. R., Claussen, M. J. 2012, ApJ, 751, 69
- Spitzer, L., 1954, ApJ, 120, 1S
- Tielens, A. G. M. M., Hollenbach, D. 1985, ApJ 291, 772
- Tielens, A. G. G. M. 2005, The Physics and Chemistry of the Interstellar Medium Cambridge, UK: Cambridge University Press
- Werner, M., Roellig, T., L., Low, F. J. et al. 2004, ApJS, 154, 1